Looking for a Cosmological Constant with the Rees-Sciama Effect

Robert G. Crittenden and Neil Turok
Joseph Henry Laboratory, Princeton University, Princeton, New Jersey 08544
(Received 16 October 1995)

In models with a cosmological constant a significant component of the cosmic microwave background (CMB) anisotropy is produced at rather low redshifts $z \lesssim 1$. In these models, the gravitational potential perturbations begin to evolve at late times, shifting the frequencies of photons passing though them. Since the potential reflects the matter density, the latter should be correlated with the CMB anisotropy. We examine this correlation and discuss the prospects for using an x-ray/COBE comparison to detect a cosmological constant.

PACS numbers: 98.80.Es, 95.85.Nv, 98.35.Ce, 98.70.Vc

The idea of a cosmological constant (Λ) has been a recurring one ever since Einstein first proposed it [1]. Recent motivations for a nonzero Λ include easing the "age crisis," reconciling dynamical measures of the matter density with prejudices for flatness, and increasing the power in large scale perturbations [2]. But rather than simply introducing another free parameter, it is more interesting to ask whether there are specific observational signals that could confirm or refute the hypothesis of a nonzero Λ.

We propose one such test here, which uses the fact that a Λ term causes the Newtonian potential Φ to start evolving at late times, producing a significant amount of cosmic microwave background (CMB) anisotropy [3]. Since Λ comes to dominate rather suddenly, this effect is most important at rather modest redshifts. But if observations of the density field allow us to reconstruct the local potential, then this should be correlated with the microwave sky. Measuring this correlation thus would constrain Λ.

The strongest present observational constraint on Λ is that from gravitational lensing, which results from the fact that if there were a large cosmological constant, then lensing events would be seen more frequently than they are. The handful of lensing events that have been observed constrains the fraction of the critical density contributed by Λ to be $\Omega_\Lambda < 0.7$ [4]. This constraint, however, is sensitive to how well the mass distributions of early type galaxies are modeled and relies on the assumption that no lensing events are obscured by dust. Other probes of Λ, such as measurements of the deceleration parameter q_0, give weaker constraints [5]. Whether our test becomes competitive with these remains to be seen, but the types of biases in the various tests are so different that it is worth exploring them all.

In the approximation of instantaneous recombination, the microwave anisotropy in a direction $\mathbf{n}$ on the sky is given by the formula

$$\frac{\delta T}{T}(\mathbf{n}) = \left[\frac{1}{4}\delta_\gamma + \mathbf{v}\cdot\mathbf{n} + \Phi\right]_i^f + 2\int_i^f d\tau\,\dot{\Phi}(\tau, \mathbf{n}(\tau_0 - \tau)) \qquad (1)\,.$$

The integral is over the conformal time τ, $\tau_f = \tau_0$ being today and τ_i being recombination. The first term represents the perturbations on the surface of last scattering, namely the perturbation to the density of the radiation-baryon fluid (δ_γ), the Doppler term ($\mathbf{v}\cdot\mathbf{n}$), and the Newtonian potential. The second term, usually called the Integrated Sachs-Wolfe (ISW) term, represents the effect of a time varying gravitational potential along the line of sight. Heuristically, it represents the redshifting of photons which must "climb out" of a different potential than they "fell into." This is called the Rees-Sciama effect [6].

In a flat, matter dominated universe, with linear growing density perturbations, Φ is constant and there is no Rees-Sciama effect. Nonlinear gravitational collapse does lead to anisotropies on very small angular scales, but of small amplitude [7]. In a universe with a significant cosmological constant, however, Φ becomes time dependent even in linear theory and an appreciable amount of anisotropy can be created at quite modest redshifts.

As Λ increases, it comes to dominate the energy density at earlier and earlier redshifts. The effect on the evolution of the potential is thus more pronounced, as is the corresponding anisotropy generated at late times. For smaller values of Λ the opposite is true; the correlated anisotropy is less, but it is more concentrated at very late epochs. As an aside, we should note that the Λ also has an indirect effect on the degree scale anisotropy, because in a flat universe the presence of Λ alters the matter-radiation balance at last scattering. In contrast, the large scale Rees-Sciama effect is independent of physics at high redshifts (e.g., reionization).

To quantify this, we expand the sky temperature in the usual spherical harmonics

$$\frac{\delta T}{T}(\mathbf{n}) \equiv \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi)\,, \qquad (2)$$

where in an isotropic ensemble the a_{lm} obey $\langle a_{lm} a_{l'm'}\rangle = \delta_{ll'}\delta_{mm'}C_l$, with C_l the angular power spectrum. An idea of how much anisotropy is produced from the late time evolution of Φ is obtained by computing the contribution to each C_l by the ISW integral prior to some redshift

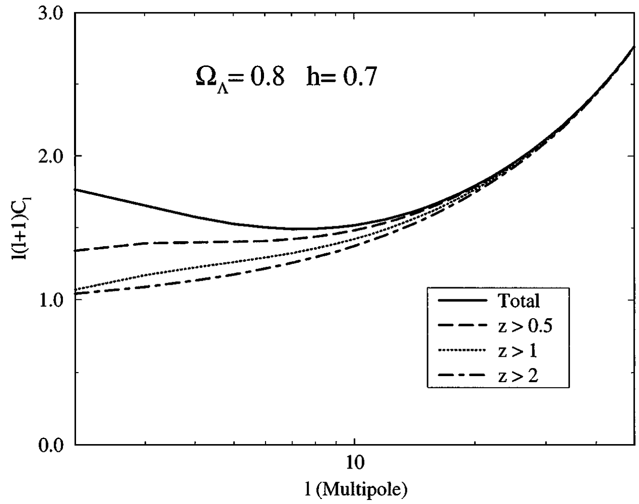

FIG. 1. The large scale anisotropy power spectrum, $C_l = \langle |a_{lm}|^2 \rangle$, for a model with $\Omega_\Lambda = 0.8$ and $h = 0.7$. Also shown is the anisotropy that is produced prior to a given redshift for $z = 0.5, 1$, and 2. A significant portion of the anisotropy is produced rather recently.

z_c. This is shown in Fig. 1. From this we see that a significant fraction of the C_l's at low l are produced at $z \lesssim 1$. (The rise of the spectrum at low l has been discussed before [8,9]; its value as a signature of Λ is limited by cosmic variance.)

Since part of the CMB anisotropy is associated with the gravitational potential at low redshift, it must be correlated with the matter distribution in our vicinity. The gravitational potential is determined from the matter distribution by Poisson's equation $\nabla^2 \Phi = 4\pi G a^2 \delta_M \rho_M$, where δ_M is the fractional density perturbation in the matter and ρ_M is the background matter density.

It is convenient to treat this in Fourier space, so that for example $\Phi(\mathbf{x}, \tau) = \sum_{\mathbf{k}} \Phi(\mathbf{k}, \tau) e^{i\mathbf{k}\cdot\mathbf{x}}$, and also to refer the density perturbation to the present time τ_0. In the matter dominated epoch, all k modes grow at the same rate, and from Poisson's equation one infers that $\Phi(\mathbf{k}, \tau) = g(\tau)\delta_M(\mathbf{k}, \tau_0)/k^2$, where $g(\tau)$ is independent of k. Inserting this in relation (1), and expanding the plane wave in spherical Bessel functions one finds

$$a_{lm}^{\mathrm{RS}} = 8\pi i^l \sum_{\mathbf{k}} Y_{lm}^*(\Omega_{\mathbf{k}}) \frac{\delta_M(\mathbf{k}, \tau_0)}{k^2} \int d\tau \dot{g}(\tau) j_l(k\Delta\tau), \tag{3}$$

where $\Delta\tau = \tau_0 - \tau$. This equation has a simple interpretation in real space (RS): It says that the RS contribution to a_{lm} comes from convolving the matter density $\delta_M(\mathbf{x}, \tau_0)$ perturbation in our vicinity with a spatial weighting function $f_l(r) Y_{lm}^*(\Omega_{\mathbf{x}})$. That is, if we substitute the inverse Fourier transform, we find

$$a_{lm}^{\mathrm{RS}} = \int d^3\mathbf{x} f_l(r) \delta_M(\mathbf{x}, \tau_0) Y_{lm}^*(\Omega_{\mathbf{x}}), \tag{4}$$

where

$$f_l(r) = \int d\tau \dot{g}(\tau) \int \frac{dk}{\pi^2} j_l(kr) j_l(k\Delta\tau). \tag{5}$$

The integral is straightforwardly performed, with the result that

$$f_l(r) = \frac{2^{2l+2}}{(2l+1)} \int d\tau \dot{g}(\tau) \frac{(r\Delta\tau)^l}{(r + \Delta\tau + |r - \Delta\tau|)^{2l+1}}. \tag{6}$$

Equations (5) and (6) tell one how to compute the Rees-Sciama contribution to each a_{lm}. The asymptotics of f are easily read off: As $r \to 0$, $f \to$ const, and as $r \to \infty$, $f \sim r^{-(l+1)}$. More importantly, $f_l(r)$ is reasonably described by a very simple approximation: For large l (we shall only be interested in $l > 2$) the second term in the integral is approximately a delta function $\delta(\Delta\tau - r)$, and the integral is approximately

$$f_l(r) \simeq \frac{2}{l(l+1)} \dot{g}(\tau_0 - r), \tag{7}$$

i.e., it is proportional to the rate of change of the local gravitational potential. We have checked that this is a reasonable approximation down to $l = 2$. Figure 2 shows $\dot{g}(z) = \dot{g}(\tau_0 - \tau(z))$ for a range of values for Λ. Note that $f_l(r)$ is independent of the power spectrum of primordial density perturbations. The only assumption needed is that the perturbations are in the pure growing mode.

The observed a_{lm}'s will differ from the Rees-Sciama result, however, because a significant component of the observed anisotropy is produced on the last scattering surface. The latter acts to obscure the correlation between the observed anisotropy and the local density fluctuations. What sort of signal-to-noise ratio may we ultimately expect in the cross correlation of the density and anisotropy, given that we are limited by cosmic variance?

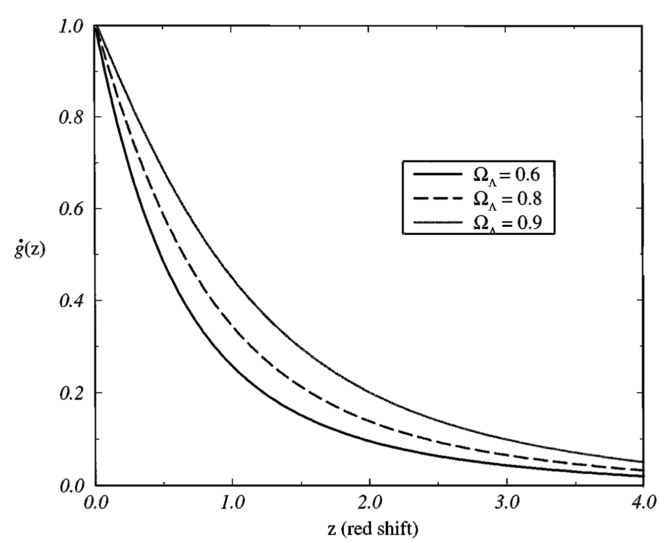

FIG. 2. The ideal weighting function $\dot{g}(z)$ as a function of redshift. Even for very large Λ, significant contributions result from low redshift, though contributions begin at higher redshifts.

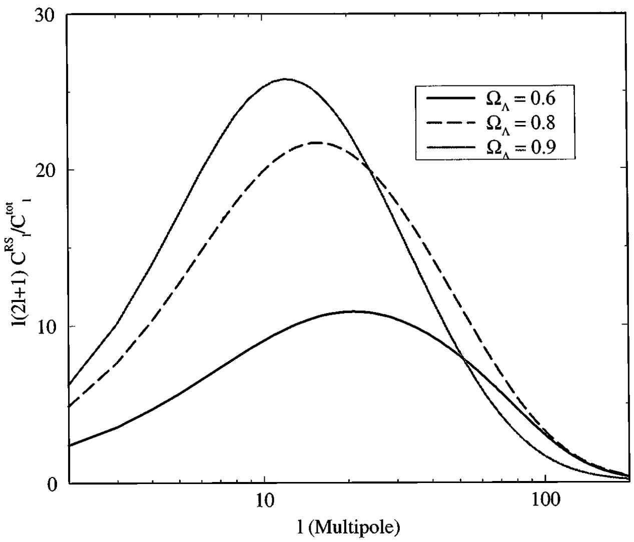

FIG. 3. The signal-to-noise ratio squared as a function of l, where the area under the curve represents the contribution for a given logarithmic interval.

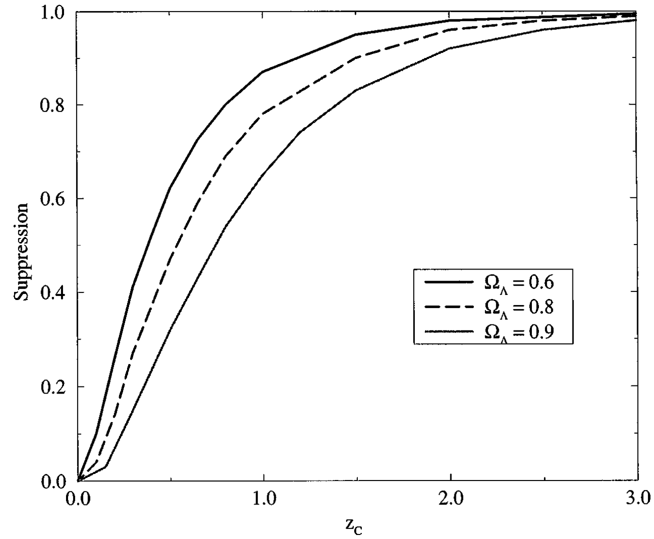

FIG. 4. We plot the reduction of the signal-to-noise ratio if the density survey is cut off beyond a given redshift.

Let us begin with the most optimistic assumption, that we have a complete survey of some tracer of the matter distribution, deep enough to see all redshifts where the cosmological constant was significant. We wish to compare the hypothesis that the a_{lm}^{RS}'s defined by Eq. (4) *are* correlated with the observed a_{lm}'s as predicted by the Λ models, with the hypothesis that they are not correlated at all. The relative likelihood of the two hypotheses can be computed for any given data set; if the correlations are real, then the expected value of this is

$$\mathcal{P} = \prod_{l,m}\left(1 - \frac{\langle a_{lm}^{\mathrm{RS}} a_{lm}^{\mathrm{tot}*}\rangle^2}{C_l^{\mathrm{RS}} C_l^{\mathrm{tot}}}\right)^{-1}. \tag{8}$$

(For a set of independent observables, $\mathcal{P}$ is the product of the individual $\mathcal{P}$'s). Defining the signal-to-noise ratio squared as $\ln(\mathcal{P})$ we infer that

$$\left(\frac{S}{N}\right)^2 \equiv \ln\mathcal{P} \geq \sum_l (2l+1)\frac{\langle a_{lm}^{\mathrm{RS}} a_{lm}^{\mathrm{tot}*}\rangle^2}{C_l^{\mathrm{RS}} C_l^{\mathrm{tot}}}. \tag{9}$$

This sum converges quickly beyond $l \sim 50$, yielding $S/N \geq 5.5, 7.4$, and 7.9 for $\Omega_\Lambda = 0.6$, 0.8, and 0.9, respectively. Figure 3 shows the contribution to this sum as a function of l. Note that the Rees-Sciama contribution is almost uncorrelated with the remainder of the anisotropy, so that $\langle a_{lm}^{\mathrm{RS}} a_{lm}^{\mathrm{tot}*}\rangle \approx C_l^{\mathrm{RS}}$.

Realistic surveys, however, are not likely to probe the density this deeply. For a survey which is less than ideal, we can get some feel for the loss in signal by considering the case where the convolution function $f_l(r)$ is the ideal one out to some cutoff redshift z_c and zero beyond. The signal-to-noise ratio in the correlation for a given multiple is then

$$\frac{S}{N} \geq \frac{\langle a_{lm}^{\mathrm{part}} a_{lm}^{\mathrm{tot}*}\rangle}{\sqrt{C_l^{\mathrm{part}} C_l^{\mathrm{tot}}}} \approx \frac{\langle a_{lm}^{\mathrm{part}} a_{lm}^{\mathrm{RS}*}\rangle}{\sqrt{C_l^{\mathrm{part}} C_l^{\mathrm{RS}}}} \frac{S^{\mathrm{ideal}}}{N} \tag{10}$$

The suppression factor is given by

$$\frac{\langle a_{lm}^{\mathrm{part}} a_{lm}^{\mathrm{RS}*}\rangle}{\sqrt{C_l^{\mathrm{part}} C_l^{\mathrm{RS}}}} = \frac{\int k^2 dk f_l(k)\tilde{f}_l(k)P_k}{\sqrt{\int k^2 dk f_l(k)^2 P_k \int k^2 dk \tilde{f}_l(k)^2 P_k}} \tag{11}$$

where

$$f_l(k) = \int r^2 dr j_l(kr)\dot{g}(r),$$
$$\tilde{f}_l(k) = \int_0^{z_c} r^2 dr j_l(kr)\dot{g}(r), \tag{12}$$

and $P_k \equiv \langle|\delta_M(k,\tau_0)|^2\rangle$. We have performed these integrals numerically and find that the result is very weakly dependent on l. The resulting suppression factor for $l = 10$ as a function of the redshift is shown in Fig. 4. As can be seen, there is a substantial signal even when the survey is cut off at rather modest z_c.

To predict the a_{lm}^{RS}, one requires a measure of the density contrast δ_M in our vicinity. Traditionally, it is assumed that this is at least roughly proportional to the fluctuation in the number density $n(\mathbf{x})$ of galaxies (or other tracers): $(\delta n/\bar{n}) = b(\delta\rho/\bar{\rho})$ where b is a "bias" factor which could depend on redshift. The dimensionless cross correlation between the RS anisotropy level predicted from a survey of mass tracers and the detected CMB anisotropy, i.e., $\langle a_{lm}^{\mathrm{pred}} a_{lm}^{\mathrm{det}}\rangle/\sqrt{C_l^{\mathrm{pred}} C_l^{\mathrm{det}}}$, is independent of b if b is constant, but it does depend on the variation of b with redshift. However, the net effect is to alter the effective weighting function $f_l(z)$, and we have seen that the cross correlation is fairly insensitive to this. A real data analysis could set limits on the variation of $b(z)$

and on Λ, and might be used to determine the magnitude of b, should a correlation be found.

Possible tracers of the mass distribution at $z \sim 1$ include radio galaxies and quasars, and a number of large scale surveys of these are underway. More immediately, it would be very interesting to correlate an all-sky x-ray survey like ROSAT with the COBE anisotropy measurement. The x rays with energies on the order of a keV appear to be consistent with a simple model in which they are all produced by active galactic nuclei (Seyfert galaxies and quasars). Surveys of five deep fields to find these active galactic nuclei indicate that their distribution in redshift (i.e., $d\bar{N}/dz$) is approximately flat for $0.5 < z < 20$ and cuts off rapidly thereafter [10], so they do indeed sample the redshift range of interest. To estimate the expected correlation, however, we need to translate this into an effective weighting function $f_l(r)$.

At any frequency, the intensity of the x-ray sky in a given direction is $\iota(\mathbf{n}) = \int \mathcal{F}(z) dN(r\mathbf{n}, z)$, where $\mathcal{F}(z)$ is the mean flux from a source at redshift z and $dN(r\mathbf{n}, z)$ is the number of sources in the redshift interval $[z, z + dz]$. [Here $r = \tau_0 - \tau(z)$.] We can express dN as $dN(r\mathbf{n}, z) = d\bar{N}[1 + b(z)\delta_M(r\mathbf{n}, z)]$, where $d\bar{N}$ is the mean value of dN. We then obtain

$$\delta\iota(\mathbf{n}) = \int dz \frac{d\bar{N}}{dz} \mathcal{F}(z) b(z) D(z) \delta_M(r\mathbf{n}, z = 0), \quad (13)$$

where $D(z)$ is the matter growth factor normalized to unity today. Comparing this with Eq. (4), we can identify

$$r^2 f_l(r) \propto b(z) \frac{dz}{d\tau} \frac{d\bar{N}}{dz} D(z) \mathcal{F}(z), \quad (14)$$

thus giving us an expression for the actual experimental weighting function. Using a simple fit to the $d\bar{N}/dz$ given in [10], and the naive assumptions that $b(z)$ and $\mathcal{F}(z)$ are constant, we find a suppression factor of ~ 0.8 for an $\Omega_\Lambda = 0.8$ universe. Barring other sources of noise, a substantial signal should be visible in the COBE-ROSAT correlation, at least for this model.

Very recently, the large angular scale fluctuations in the ROSAT survey have been studied, with the finding that there is a significant autocorrelation on scales $\theta < 6^o$ [11]. The cross-correlation function is less susceptible to noise than the autocorrelation function; this is illustrated by the analysis of the FIRS experiment [12], where the cross correlation with COBE was measured even though the autocorrelation was insignificant. In fact, an upper bound to the COBE/x-ray correlation has already been found by Bennett *et al.* [13] using the COBE first year maps and the HEA0 1 A-2 x-ray map, but it is too weak to give an interesting limit on Λ. Correlating ROSAT with the COBE four year maps should provide a much stronger limit.

In this paper, we have focused on Λ models and found a significant correlation between local density perturbations and the CMB anisotropy. However, we wish to emphasize that *some* correlation is expected in most cosmological models, such as those involving a spatially open universe or cosmic defects. The time independence of the Newtonian potential in the flat matter dominated universe is very much a special case. While the observation of a correlation would not uniquely single out Λ as its explanation, the absence of a correlation would impose a powerful constraint on many models.

We thank Jim Peebles for suggesting the use of ROSAT for this test, and Ue-Li Pen and Ed Turner for helpful discussions. This work was partially supported by NSF Contract No. PHY90-21984, and the David and Lucile Packard Foundation.

[1] A. Einstein, Sitzungsber. Preuss. Akad. Wiss. **142** (1917).
[2] J.P. Ostriker and P.J. Steinhardt, Penn Report No. UPR-0659T, astro-ph/9505066; L.M. Krauss and M.S. Turner, Report No. CWRU-P6-95, astro-ph/9504003; G. Efstathiou, W.J. Sutherland, and S.J. Maddox, Nature (London) **348**, 705 (1990); P.J.E. Peebles, Astrophys. J. **284**, 439 (1984).
[3] L.A. Kofman and A.A. Starobinskii, Sov. Astron. Lett. **11**, 271 (1985).
[4] D. Maoz and H. Rix, Astrophys. J. **416**, 425 (1993).
[5] For a recent review, see S.M. Carroll, W.H. Press, and E.L. Turner, Annu. Rev. Astron. Astrophys. **30**, 499 (1992).
[6] M.J. Rees and D.W. Sciama, Nature (London) **217**, 511 (1968).
[7] U. Seljak, Report No. MIT-CSR-95-13, astro-9506048; R. Tuluie, P. Launa, and P. Anninos, Report No. CGPG-95-96-10-1, astro-ph.510019.
[8] N. Sugiyama and J. Silk, Phys. Rev. Lett. **73**, 509 (1994).
[9] E. Bunn and N. Sugiyama, Astrophys. J. **446**, 49 (1995).
[10] A. Comastri, G. Setti, G. Zamorani, and G. Hasinger, Astron. Astrophys. **296**, 1 (1995).
[11] A.M. Soltan, G. Hasinger, R. Egger, S. Snowden, and J. Trumper, Max Planck report, 1995 (to be published).
[12] K. Ganga *et al.*, Astrophys. J. Lett. **410**, L57 (1993).
[13] C. Bennett *et al.*, Astrophys. J. Lett. **414**, L77 (1993).